Math 322 - Real Analysis II - Directed Inquiry - Spring 2010
Instructor: Jeff Hamrick
Course Syllabus

ON ANALYSIS. When most mathematicians think of "analysis," they think of "rigorous calculus." Analysis is the study of a collection of concepts—limits, sequences, continuity, differentiability, and integrability—that are often linked to real-valued or complex-valued functions. Arguably, ancient mathematicians were the first true analysts. Zeno discredited the notion of an infinitesimal quantity by stating several apparent paradoxes that would not be unraveled for hundreds of years. Eudoxus and Archimedes used calculus-like techniques to compute the areas and volumes of regions and solids. Bhaskara, an Indian mathematician, effectively proved Rolle's Theorem in the 12th century.

In the 14th century another Indian, Madhava of Sangamagrama, more or less developed the idea of expanding a function into a sum of other functions, like polynomials, sines, and cosines. Two hundred years later, Newton and Leibniz independently developed modern calculus by articulating and proving the Fundamental Theorem of Calculus which, roughly speaking, says that differentiation and integration are operations that undo one another. Over time, analysis developed numerous important sub-branches including the calculus of variations, ordinary differential equations, partial differential equations, Fourier analysis, and the theory of generating functions.

Mathematicians had tremendous difficulty developing the concepts that we will study this semester. It took a concerted effort by Euler (to make it clear what we actually mean when we say "function"), Cauchy (to standardize haphazard notation), and Bolzano (to pin down an appropriate definition of continuity) to place modern analysis on solid mathematical footing. Pathological counterexamples generated by mathematicians like Weierstrass forced the mathematical community to think very carefully about certain definitions. Later, Riemann clearly established the connection between the area problem and the Fundamental Theorem of Calculus as understood by Newton and Leibniz. His work was later extended by Stieltjes and, still later, was further extended by Lebesgue.

Today, analysis remains (along with algebra) one of the two main branches of mathematical thought. Modern analysis has only been around for about 400 years, but its incredible success has furnished humanity with the intellectual capital needed to foment both the industrial and information revolutions of the 18th and 21st centuries, respectively. Analysis is one of humankind's most important accomplishments over the past 2000 years. I hope you'll enjoy learning a little bit about it with me this semester.

ON COURSE GOALS. After you are done with this directed inquiry, you should

- Understand basic definitions and important theorems (and counterexamples) related to the continuity, differentiability, and Riemann-Stieltjes integrability of functions whose domain and codomain are the real numbers;
- Be familiar with the Moore method of teaching and learning mathematics;
- Be able to use axioms and definitions to investigate examples, produce counterexamples, and prove lemmas and theorems on your own;
- Be able to re-situate your knowledge of calculus in a far more rigorous, axiomatic framework;
- Be able to critique proofs written by other people;

- Be confident discussing your mathematical insights and struggles at a chalkboard with one or more of your colleagues;
- Be comfortable writing mathematical arguments in a clear, concise, and persuasive fashion; and
- Be somewhat closer to attaining the elusive status known as "mathematical maturity."

In general, this course will prepare successful students for further undergraduate courses in abstract mathematics (topology, complex analysis, abstract algebra, combinatorics) and graduate-level courses in measure theory, differential equations, complex analysis, probability theory, and statistics.

ABOUT ME. My name is Jeff Hamrick. I'm an assistant professor in the Department of Mathematics and Computer Science at Rhodes College. Please call me Jeff. My office is located in room 318 of Ohlendorf Hall. I will hold office hours from 11:00 a.m. - 12:00 noon on Mondays, 3:00 p.m. - 4:00 p.m. on Wednesdays, and 1:30 - 4:30 p.m. on Thursdays during the spring semester. My office number is 901/843-3253 and my e.mail address is `hamrickj@rhodes.edu`. Please stop by my office anytime. If you're unable to make my office hours, let me know and we may be able to schedule an appointment at an alternate time.

ABOUT YOU. You should be hard-working and enthusiastic about learning! This course features a fairly traditional treatment of certain topics in mathematical analysis, so you should already have taken Math 121, Math 122, Math 201, and Math 321 (or the equivalent courses) at another academic institution.

ABOUT US. We will meet to talk about mathematical analysis at least once per week (for one hour) at a time that is mutually convenient. We will primarily use my own notes for this directed inquiry, but we will also use a number of real analysis textbooks as references in this course, including Rudin's *Principles of Mathematical Analysis, 3rd ed.* (ISBN: 978-0070542358) and Lang's *Undergraduate Analysis* (ISBN: 978-0387948416).

ON THE COURSE NOTES. Because I consider myself an analyst who happens to dwell in the realms of statistics and mathematical finance, I have worked hard over the years to develop a comprehensive set of notes covering the analysis of the real line and functions defined on that real line. My notes, which I am providing to you at no charge, cover the following topics: preliminary notions from logic and set theory, sequences, series, continuity, differentiability, Riemann and Stieltjes integrals, and pointwise/uniform/L^p convergence of sequences of functions.

This semester, your task is to work through as many definitions, examples, counterexamples, theorems, and lemmas as possible on the following topics: continuity, differentiability, and Riemann-Stieltjes integration. You must prove, write expositions of, or otherwise illustrate a sizeable number of examples, counterexamples, theorems, and lemmas (axioms and definitions do not count for credit!) in order to earn a good grade in this directed inquiry.

ON YOUR NOTEBOOK. Keep all of your formal write-ups of examples, theorems, lemmas, and counterexamples in a three-ring binder. Periodically (probably weekly) turn in your work. I will grade your work and mark some of it "approved" after offering comments and suggestions for improvement. Occasionally, I will reject one or more of your write-ups and insist that you re-formulate (or scrap) your work.

ON MIDTERM EXAMINATIONS. There are no traditional midterm examinations in this course. Your objective is to work through (and write up carefully) as many elements of the course notes as possible.

ON THE FINAL EXAMINATION. There is no traditional final examination in this course. Your grade in this course will be computed according to the following rule:

Number of Correct Items from Course Notes	Grade
greater than 138	A
between 117 and 137	B
between 83 and 116	C
between 62 and 82	D
less than 62	F

In order to be eligible for a particular grade with n successfully proven/illustrated/expounded examples/theorems/lemmas, at least $[n/4]$ of those examples/theorems/lemmas must be located in the sections of the course notes dealing with continuity, differentiability, and Riemann-Stieltjes integration, respectively. Pluses and minuses shall be awarded at my discretion based on exceptionally good (or mediocre) work within some grade level.

ON CHEATING. In this class, we will adhere to the provisions of the Rhodes College Honor Code. In general, I encourage you to work on the course notes with colleagues, including faculty members and students who may or may not have taken real analysis. You should disclose all forms of collaboration in your write-ups. **I absolutely forbid the practice of looking up proofs on the Internet or in textbooks other than the two textbooks that are authorized references for this course. Many theorems in real analysis are quite famous and proofs of them can be found without too much difficulty. You must work and struggle on your own.**

In general, if the Rhodes College Honor Council concludes that a student in this course has violated the honor code, I will adhere to the recommendations of the honor council. However, I reserve the right to lower a student's grade in this course if I sincerely believe that an infraction has occurred.