

**Syllabus**  
**Math 482, Section 1**  
**CRN 12402**  
**Fall 2011**

**Instructor:** Eric Gottlieb  
**Meetings:** MWF 1:00 – 1:50 in 225 Ohlendorf  
**Text:** *Introduction to Combinatorics*, by Erickson  
**Office Hours:** MWF 2 – 3 and TR 11 – 12 (T by appt only) in 317 Ohlendorf  
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**The subject:** Combinatorics is the study of sets, usually but not always finite, endowed with some kind of structure. For example, an ordering of the numbers 1 through  $n$  is the set of permutations. This set can be endowed with the algebraic structure of a group or the combinatorial structure of a partially ordered set. There are also natural ways to view the set of permutations as a graph.

There are many other heavily studied combinatorial structures, including other examples of groups, partially ordered sets, and graphs, as well as directed graphs, matroids, Latin squares, integer partitions, set partitions, codes, block designs, and many others. Some of these have important applications to areas such as circuit design, computing, experimental design, phylogeny, transportation, and telecommunications. All are of interest in their own right.

A wide and growing range of techniques are used in combinatorics. Some of these, such as the sum and product principles, are straightforward. Others, like the pigeonhole principle, seem simple but can be applied in subtle ways to prove interesting and hard theorems. Still others are more sophisticated, such as generating functions, recurrence relations, Burnside's lemma, and Pólya counting.

There are three essential questions that arise in combinatorics, though not every topic fits neatly into one of these categories. The first is that of existence: is there a finite set with some specified structure? For example, can there be a party of six people so that there are neither three people who know each other nor three people who are strangers to each other? The answer in this case is no, but this is not immediately obvious unless you are familiar with Ramsey's theorem.

The second main question asked in combinatorics concerns the number of arrangements with the desired structure. Ideally, we seek an answer given by a nice formula. For example, there are  $n!$  permutations (i.e., total orders) on  $n$  letters. Unfortunately, many combinatorial structures are not counted so cleanly. One can then proceed by developing bounds on the number of objects in question, by approximating the number when the problem is sufficiently large, or by finding a different set of objects that is in bijection with the set of objects of interest.

The last big question is that of construction. How does one generate a uniformly random permutation? Issues of this type are computational in nature and hence are concerned not just with the correctness of a given algorithm but also with its efficiency.

It is impossible for any text or course on combinatorics to be comprehensive; one could spend one's entire life studying just the properties of permutations. This class will offer glimpses of combinatorics through selected techniques, theorems, applications, and open questions.

In most mathematical areas, such as analysis and algebra, it is typically necessary to learn a great deal of mathematics in order to even understand what a question is asking. In contrast, research level questions in combinatorics can often be posed in a few minutes to a bright high school student. The abundance of easily-stated, hard-to-answer questions is an attractive characteristic of combinatorics.

**Your mathematical background** should include the material covered in Transition to Advanced Mathematics. Exposure to linear algebra and group theory would be nice but is not essential. This material will be discussed as needed.

**Material to be covered** will depend on the rate at which we proceed. In the first two thirds of the semester, I plan to cover most of the sections from Chapters 1 – 6. In the last third, I will talk about some identities involving (number) partitions and Fibonacci (and related) numbers.

**Homework:** I will assign and collect homework roughly every week or two. The grade you receive on this work will depend on the clarity of your writing as well as the correctness of your mathematics. Partial solutions, computer programs, and conjectures are welcome and may receive partial credit. Proofs of more general results may receive extra credit. LaTeX is a type of mathematical typesetting software. I will be happy to help you to learn how to create documents in LaTeX, and I will award 5% extra credit on each homework assignment completed in LaTeX.

**Exams:** There will be two midterm exams and a cumulative final exam as indicated. The dates are fixed, but the material to be covered is tentative and depends on our pace.

Exam	Date	Material to be covered
MT 1	Friday 23 September	Chapters 1 – 3
MT 2	Friday 28 October	Chapters 4 – 6
Final	Saturday 10 December at 8:30 AM (121-2) Monday 12 December at 1:00 PM (482-1) Tuesday 13 December at 1:00 PM (121-1)	Comprehensive, with extra emphasis on material not covered on earlier exams

**Your Final Score and Grades** are determined as follows:

Homework Average: 23%  
Midterm Exams: 23%  
Final Exam: 31%

The letter equivalent of your number grade will be assigned according to the following scale. These represent minimum grades in order to allow me some discretion. It is possible, for example, to receive a B while earning a total score of less than 83%. However, if you earn a score between 83% and 86%, you are guaranteed to receive a grade of B or better.

93-100	90-92	87-89	83-86	80-82	77-79	73-76	70-72	67-69	63-66	60-62	<59
A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F

**The Honor Code:** All graded work must comply with the Rhodes Honor Code.